Some Solved Problems for Week One

1.1#1 Which are propositions?

(a) Lemuel Harrington, of Burbank, CA, was President of the United States on July 18, 1897.

(b) $x/x = 1$

(c) $13 + 24 = 35$

(d) "The Star Spangled Banner" was played on that occasion.

(e) x is positive, negative, or zero.

(f) If x is real, then x is positive, negative, or zero.

Solution: (a) is a false proposition. (b) is not a proposition, since we know nothing about x. (c) is a false proposition. (d) is not a proposition, since we don't know what occasion this refers to. (e) is not a proposition, since we know nothing about x. (f) is a true proposition.

1.1#4 We are looking into a system in which a statement is defined to be a string of five letters. A statement is a proposition if it is listed in a specified English-language dictionary. There is one axiom: *groan*. Given the proposition P, P is a proposition if and only if there is a sequence $P_1, P_2, \ldots, P_k$ of theorems in which P_k is P.

(a) Prove *cloth*.

Solution: Here is one solution: groan, grown, brown, blown, blows, blots, clots, cloth.

(b) How would we demonstrate that a particular proposition is false?

Solution: This is harder than I made it sound in class. I was thinking of a special case. Suppose that P is the proposition in question. In the general case, one must show that *there is no sequence from P to any theorem*. In the special case that I had in mind, P was isolated, i.e., none of the 125 possible first steps led from P to a theorem. An algorithm for the general case would be more complex. Happily, we don't have to provide one.

1.3#1 Let P be "Howard fell" and let Q be "Howard broke his leg."

(a) Write English sentences corresponding to each:

i. $P \wedge Q$ is "Howard fell and broke his leg."

ii. $\neg P \wedge \neg Q$ is "Howard neither fell nor broke his leg."

iii. $\neg(P \wedge Q)$ is (prior to massaging) "It is not the case that Howard fell and broke his leg."

iv. $Q \vee \neg P$ is "either Howard broke his leg, or Howard didn't fall."

1.1#2 One Tuesday morning, your friend says, "If today is Wednesday, then today is Thursday." Discuss the truth value of your friend's comment.

Solution: It's true, since the comment was made on a Tuesday.

1.3#5 Suppose we are shown four cards that are lying on a table. We know that one side of each card is labeled with a letter, the other with a number. The cards appear as follows:

1	C	B	2

We are told that if any card bears a B on one side then it must bear a 2 on the other side. Which cards need not be turned over to either verify or refute this claim? Explain.

Solution: We need not turn the second card over, since it does not show a B and so it doesn't matter to us what's on the other side, nor must we turn the fourth card over, since it shows a 2 and so we don't care what letter is on the other side.

1.3#11 We are to exhibit truth tables for five statement forms. Part (a) is a tautology. Part (d) is, too, since it proves the logical equivalence of an implication to its contrapositive. Part (e) was done in class. Here are verbal descriptions of the other two:

(b) $P \Rightarrow (P \Rightarrow P)$ is trivially true, since $(P \Rightarrow P)$ is a tautology.

(c) $(P \Rightarrow P) \Rightarrow P$ is true when P is true, false when P is false.

1.3#17 We are to use the definition of $P \Rightarrow Q$ as an abbreviation for $\neg(P \wedge \neg Q)$ to rewrite $P \Rightarrow (Q \Rightarrow (R \Rightarrow S))$ using only $\neg$ and $\wedge$ as connectives.

Solution: Without simplifying, we get

$$\neg(P \wedge \neg\neg(Q \wedge \neg\neg(R \wedge \neg S))).$$